M22/5/MATHY/HP1/ENG/TZ2/XX/M

Markscheme

May 2022

Mathematics: applications and interpretation

Higher level

Paper 1

24 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**.
A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.
R Marks awarded for clear **Reasoning**.
AG Answer given in the question and so no marks are awarded.
FT Follow through. The practice of awarding marks, despite candidate errors in previous parts, for their correct methods/answers using incorrect results.

Using the markscheme

1 General

Award marks using the annotations as noted in the markscheme *eg* ***M1, A2***.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any.
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***A3***, ***M2*** *etc*., do **not** split the marks, unless there is a note.
- The response to a "show that" question does not need to restate the ***AG*** line, unless a **Note** makes this explicit in the markscheme.
- Once a correct answer to a question or part question is seen, ignore further working even if this working is incorrect and/or suggests a misunderstanding of the question. This will encourage a uniform approach to marking, with less examiner discretion. Although some candidates may be advantaged for that specific question item, it is likely that these candidates will lose marks elsewhere too.
- An exception to the previous rule is when an incorrect answer from further working is used **in a subsequent part**. For example, when a correct exact value is followed by an incorrect decimal approximation in the first part and this approximation is then used in the second part. In this situation, award ***FT*** marks as appropriate but do not award the final ***A1*** in the first part. Examples:

	Correct answer seen	**Further working seen**	**Any FT issues?**	**Action**
1.	$8\sqrt{2}$	$5.65685...$ *(incorrect decimal value)*	No. Last part in question.	Award ***A1*** for the final mark *(condone the incorrect further working)*
2.	$\frac{35}{72}$	$0.468111\ldots$ (incorrect decimal value)	Yes. Value is used in subsequent parts.	Award ***A0*** for the final mark *(and full **FT** is available in subsequent parts)*

3 Implied marks

Implied marks appear in **brackets e.g. *(M1)***, and can only be awarded if **correct** work is seen or implied by subsequent working/answer.

4 Follow through marks (only applied after an error is made)

Follow through (***FT***) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) (e.g. incorrect value from part (a) used in part (d) or incorrect value from part (c)(i) used in part (c)(ii)). Usually, to award ***FT*** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if all the marks awarded in a subsequent part are for the answer or are implied, then ***FT*** marks should be awarded for *their* correct answer, even when working is not present.

For example: following an incorrect answer to part (a) that is used in subsequent parts, where the markscheme for the subsequent part is ***(M1)A1***, it is possible to award full marks for *their* correct answer, **without working being seen.** For longer questions where all but the answer marks are implied this rule applies but may be overwritten by a **Note** in the Markscheme.

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate.
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks, by reflecting on what each mark is for and how that maps to the simplified version.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- The markscheme may use the word "their" in a description, to indicate that candidates may be using an incorrect value.
- If the candidate's answer to the initial question clearly contradicts information given in the question, it is not appropriate to award any ***FT*** marks in the subsequent parts. This includes when candidates fail to complete a "show that" question correctly, and then in subsequent parts use their incorrect answer rather than the given value.
- Exceptions to these ***FT*** rules will be explicitly noted on the markscheme.
- If a candidate makes an error in one part but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the command term was "Hence".

5 Mis-read

If a candidate incorrectly copies values or information from the question, this is a mis-read (***MR***). A candidate should be penalized only once for a particular misread. Use the ***MR*** stamp to indicate that this has been a misread and do not award the first mark, even if this is an ***M*** mark, but award all others as appropriate.

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, $\sin\theta = 1.5$, non-integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.
- If a candidate uses a correct answer, to a "show that" question, to a higher degree of accuracy than given in the question, this is NOT a misread and full marks may be scored in the subsequent part.
- ***MR*** can only be applied when work is seen. For calculator questions with no working and incorrect answers, examiners should **not** infer that values were read incorrectly.

6 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If the command term is 'Hence' and not 'Hence or otherwise' then alternative methods are not permitted unless covered by a note in the mark scheme.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for parts of questions are indicated by **EITHER** . . . **OR**.

7 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation** for example 1.9 and $1,9$ or 1000 and $1,000$ and 1.000.
- Do not accept final answers written using calculator notation. However, ***M*** marks and intermediate ***A*** marks can be scored, when presented using calculator notation, provided the evidence clearly reflects the demand of the mark.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, some **equivalent** answers will generally appear in brackets. Not all equivalent notations/answers/methods will be presented in the markscheme and examiners are asked to apply appropriate discretion to judge if the candidate work is equivalent.

8 Format and accuracy of answers

If the level of accuracy is specified in the question, a mark will be linked to giving the answer to the required accuracy. If the level of accuracy is not stated in the question, the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures.*

Where values are used in subsequent parts, the markscheme will generally use the exact value, however candidates may also use the correct answer in subsequent parts. The markscheme will often explicitly include the subsequent values that come "*from the use of 3 sf values*".

Simplification of final answers: Candidates are advised to give final answers using good mathematical form. In general, for an ***A*** mark to be awarded, arithmetic should be completed, and any values that lead to integers should be simplified; for example, $\sqrt{\frac{25}{4}}$ should be written as $\frac{5}{2}$. An exception to this is simplifying fractions, where lowest form is not required (although the numerator and the denominator must be integers); for example, $\frac{10}{4}$ may be left in this form or written as $\frac{5}{2}$. However, $\frac{10}{5}$ should be written as 2, as it simplifies to an integer.

Algebraic expressions should be simplified by completing any operations such as addition and multiplication, e.g. $4e^{2x} \times e^{3x}$ should be simplified to $4e^{5x}$, and $4e^{2x} \times e^{3x} - e^{4x} \times e^{x}$ should be simplified to $3e^{5x}$. Unless specified in the question, expressions do not need to be factorized, nor do factorized expressions need to be expanded, so $x(x+1)$ and $x^2 + x$ are both acceptable.

Please note: intermediate ***A*** marks do NOT need to be simplified.

9 Calculators

A GDC is required for this paper, but If you see work that suggests a candidate has used any calculator not approved for IB DP examinations (eg CAS enabled devices), please follow the procedures for malpractice.

10. Presentation of candidate work

Crossed out work: If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work unless an explicit note from the candidate indicates that they would like the work to be marked.

More than one solution: Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise. If the layout of the responses makes it difficult to judge, examiners should apply appropriate discretion to judge which is “first”.

1. (a) $\left(\frac{17+25}{130}=\right)\frac{42}{130}\quad\left(\frac{21}{65},\ 0.323076\ldots\right)$ ***A1***

[1 mark]

(b) $\left(\frac{17}{17+25}=\right)\frac{17}{42}\quad(0.404761\ldots)$ ***A1A1***

Note: Award ***A1*** for correct numerator and ***A1*** for correct denominator. Award ***A1A0*** for working of $\frac{17/130}{\text{their answer to (a)}}$ if followed by an incorrect answer.

[2 marks]

(c) $\frac{41}{130}\times\frac{40}{129}$ ***A1M1***

Note: Award ***A1*** for two correct fractions seen, ***M1*** for multiplying their fractions.

$=\frac{1640}{16770}\approx 0.0978\left(0.0977936\ldots,\ \frac{164}{1677}\right)$ ***A1***

[3 marks]
Total [6 marks]

2. (a) $\sin\theta = \frac{2.1}{2.8}$ **OR** $\tan\theta = \frac{2.1}{1.85202\ldots}$ ***(M1)***

$(\theta =)\ 48.6°\ \ (48.5903\ldots°)$ ***A1***

[2 marks]

(b) **METHOD 1**

$\sqrt{2.8^2 - 2.1^2}$ **OR** $2.8\cos(48.5903...)$ **OR** $\frac{2.1}{\tan(48.5903\ldots)}$ ***(M1)***

> **Note:** Award ***M1*** for attempt to use Pythagorean Theorem with 2.1 seen or for attempt to use cosine or tangent ratio.

1.85 m (1.85202…) ***(A1)***

> **Note:** Award the ***M1A1*** if 1.85 is seen in part (a).

(6.4 – 1.85202…)
4.55 m (4.54797…) ***(A1)***

> **Note:** Award ***A1*** for 4.55 or equivalent seen, either as a separate calculation or in Pythagorean Theorem.

$\sqrt{(4.54797\ldots)^2 + 2.1^2}$
5.01 m (5.00939…m) ***A1***

METHOD 2
attempt to use cosine rule ***(M1)***
$(c^2 =)\ 2.8^2 + 6.4^2 - 2(2.8)(6.4)\cos(48.5903\ldots)$ ***(A1)(A1)***

> **Note:** Award ***A1*** for 48.5903...° substituted into cosine rule formula, ***A1*** for correct substitution.

$(c =)\ 5.01$ m $(5.00939\ldots\text{m})$ ***A1***

[4 marks]

(c) camera 1 is closer to the cash register than camera 2 (and both cameras are at the same height on the wall) ***R1***
the larger angle of depression is from camera 1 ***A1***

> **Note:** Do not award ***R0A1***. Award ***R0A0*** if additional calculations are completed and used in their justification, as per the question. Accept “1.85<4.55” or “2.8<5.01” as evidence for the ***R1***.

[2 marks]
Total [8 marks]

3. (a) $\left(\mathrm{E}(X)=\right) 10\times 0.8$ ***(M1)***

8 (people) ***A1***

[2 marks]

(b) recognition of binomial probability ***(M1)***

$0.0881 \quad (0.0880803\ldots)$ ***A1***

[2 marks]

(c) 0.8 and 6 seen **OR** 0.2 and 3 seen ***(A1)***

attempt to use binomial probability ***(M1)***

$0.121 \quad (0.120873\ldots)$ ***A1***

[3 marks]

Total [7 marks]

4. (a) **EITHER**

attempt to substitute 3, 4 and 7 into area of a trapezoid formula ***(M1)***

$(A=)\ \frac{1}{2}(7+4)(3)$

OR

given area expressed as an integral ***(M1)***

$(A=)\ \int_{-1}^{2}(6-x)\ \mathrm{d}x$

OR

attempt to sum area of rectangle and area of triangle ***(M1)***

$(A=)\ 4\times 3+\frac{1}{2}\ (3)(3)$

THEN

16.5 (square units) ***A1***

[2 marks]

(b) (i) $(A=)\ \int_{-1}^{2}1.5x^2-2.5x+3\ \mathrm{d}x$ ***A1A1***

Note: Award ***A1*** for the limits $x=-1$, $x=2$ in correct location. Award ***A1*** for an integral of the quadratic function, $\mathrm{d}x$ must be included. Do not accept "y" in place of the function, given that two equations are in the question.

(ii) 9.75 (square units) ***A1***

[3 marks]

(c) $16.5-9.75$ ***(M1)***

6.75 (square units) ***A1***

[2 marks]

Total [7 marks]

5. (a) *Accept any one of the following (or equivalent):*
one minimum and one maximum point
three x-intercepts or three roots (or zeroes)
one point of inflexion ***R1***

Note: Do not accept "S shape" as a justification.

[1 mark]

(b) (i) $(d =)\ -5$ ***A1***

(ii) $8 = a + b + c$
$4 = 8a + 4b + 2c$
$0 = 27a + 9b + 3c$ ***A2***

Note: Award ***A2*** if all three equations are correct.
Award ***A1*** if at least one is correct. Award ***A1*** for three correct equations that include the letter "d".

(iii) $a = 2,\ b = -12,\ c = 18$ ***A1***

[4 marks]

(c) equating found expression to zero ***(M1)***
$0 = 2t^3 - 12t^2 + 18t - 5$
$t = 0.358216\ldots,\ 1.83174\ldots,\ 3.81003\ldots$ ***(A1)***

(so total time in debt is $3.81003\ldots - 1.83174\ldots + 0.358216 \approx$)
$2.34\ \ (2.33650\ldots)$ years ***A1***

[3 marks]
Total [8 marks]

6. (a)

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{pmatrix}$$ ***A2***

> **Note:** Award ***A2*** for the transposed matrix. Presentation in markscheme assumes columns/rows ordered A-E; accept a matrix with rows and/or columns in a different order only if appropriately communicated.
> Do not ***FT*** from part (a) into part (b).

[2 marks]

(b) raising their matrix to a power of 5 ***(M1)***

$$\boldsymbol{M}^5 = \begin{pmatrix} 17 & 9 & 2 & 3 & 5 \\ 17 & 10 & 3 & 4 & 4 \\ 13 & 6 & 2 & 2 & 4 \\ 8 & 5 & 1 & 2 & 2 \\ 18 & 11 & 2 & 4 & 5 \end{pmatrix}$$ ***(A1)***

> **Note:** The numbers along the diagonal are sufficient to award ***M1A1***.

(the required number is $17+10+2+2+5=$) 36 ***A1***

[3 marks]
Total [5 marks]

7. **METHOD 1**

$\frac{u_1}{1-r} = 9$ ***A1***

therefore $u_1 = 9 - 9r$

$u_1 = 4 + u_1 r$ ***A1***

substitute or solve graphically: ***M1***

$9 - 9r = 4 + (9 - 9r)r$ **OR** $\frac{4}{(1-r)^2} = 9$

$9r^2 - 18r + 5 = 0$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

then $u_1 = 9 - \left(9 \times \frac{1}{3}\right) = 6$

$u_3 = 6 \times \frac{1}{3}^{2} = \frac{2}{3}$ ***A1***

METHOD 2

$\frac{u_1}{1-r} = 9$ ***A1***

$r = \frac{u_1 - 4}{u_1}$ ***A1***

attempt to solve ***M1***

$\frac{u_1}{1 - \left(\frac{u_1 - 4}{u_1}\right)} = 9$

$\frac{u_1}{\left(\frac{4}{u_1}\right)} = 9$

$(u_1)^2 = 36$

$u_1 = \pm 6$

attempting to solve both possible sequences

$6, 2, \ldots$ or $-6, -10 \ldots$

$r = \frac{1}{3}$ or $r = \frac{5}{3}$

only $r = \frac{1}{3}$ is possible as the sum to infinity exists ***R1***

$u_3 = 6 \times \left(\frac{1}{3}\right)^2 = \frac{2}{3}$ ***A1***

Total [5 marks]

8. (a) $\pi \times 2^2 \times \frac{30}{360}$ ***(M1)***

$= 1.047 \text{ cm}^2$ ***A1***

Note: Do not award the final mark if the answer is not correct to 4 sf.

[2 marks]

(b) attempt to substitute any two values from 1.5, 2.5, 25 or 35 into area of sector formula ***(M1)***

$\left(\text{upper bound} = \pi \times 2.5^2 \times \frac{35}{360} =\right)$ 1.91 cm^2 $(1.90895\ldots)$ ***A1***

$\left(\text{lower bound} = \pi \times 1.5^2 \times \frac{25}{360} =\right)$ 0.491 cm^2 $(0.490873\ldots)$ ***A1***

Note: Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers.

[3 marks]

(c) $\left(\left|\frac{1.047 - 1.90895\ldots}{1.90895\ldots}\right| \times 100 =\right)$ 45.2 $(\%)$ $(45.1532\ldots)$ ***A1***

$\left(\left|\frac{1.047 - 0.490873\ldots}{0.490873\ldots}\right| \times 100 =\right)$ 113 $(\%)$ $(113.293\ldots)$ ***A1***

so the largest percentage error is 113 % ***A1***

Note: Accept 45.1 (%) (45.1428), from use of full accuracy answers. Given the nature of the question, accept correctly rounded **OR** correctly truncated 3 significant figure answers. Award ***A0A1A0*** if 113% is the only value found.

[3 marks]

Total [8 marks]

9. (a) $\overline{x} = 4.63 \ (4.62686\ldots)$ ***A1***

[1 mark]

(b) $s_{n-1} = 1.098702$ ***(A1)***

$s_{n-1}^2 = 1.21 \ (1.207146\ldots)$ ***A1***

Note: Award ***A0A0*** for an answer of 1.19 from biased estimate.

[2 marks]

(c) (i) $H_1 : \mu > 4.4$ ***A1***

(ii) **METHOD 1**

using a z-test ***(M1)***

$p = 0.0454992\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

METHOD 2

using a t-test ***(M1)***

$p = 0.0478584\ldots$ ***A1***

$p < 0.05$ ***R1***

reject null hypothesis ***A1***

(therefore there is significant evidence that the IB HL math students know more digits of π than the population in general)

Note: Do not award ***R0A1***. Allow ***R1A1*** for consistent conclusion following on from their p-value.

[5 marks]

Total [8 marks]

10. (a) $y = \ln\left(\frac{1}{x-2}\right)$

an attempt to isolate x (or y if switched) ***(M1)***

$e^{y} = \frac{1}{x-2}$

$x - 2 = e^{-y}$

$x = e^{-y} + 2$

switching x and y (seen anywhere) ***M1***

$f^{-1}(x) = e^{-x} + 2$ ***A1***

[3 marks]

(b) sketch of $f(x)$ and $f^{-1}(x)$ ***(M1)***

$x = 2.12 \ (2.12002\ldots)$ ***A1***

[2 marks]

Total [5 marks]

11. (a) **METHOD 1** – *(With FV=4000)*

EITHER
N= 10
I=1.5
FV= 4000
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for $(3.5-2=)\ 1.5$ seen and ***M1*** for all other entries correct.

OR
$4000 = A(1+0.015)^{10}$ ***(A1)(M1)***

Note: Award ***A1*** for 1.5 or 0.015 seen, ***M1*** for attempt to substitute into compound interest formula **and** equating to 4000.

THEN
(PV =) \$3447 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

METHOD 2 – *(With FV including inflation)*
calculate FV with inflation
4000×1.02^{10} ***(A1)***
(=4875.977…)

EITHER
$4000\times 1.02^{10} = \text{PV}\times 1.035^{10}$ ***(A1)***
OR
N= 10
I= 3.5
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for *their* FV and all other entries correct.

THEN
(PV =) \$3457 ***A1***

Note: Award ***A0*** if not rounded to a whole number or a negative sign given.

continued…

Question 11 continued

METHOD 3 – *(Using formula to calculate real rate of return)*
(real rate of return =) $1.47058\ldots(\%)$ ***(A1)***

EITHER
$4000 = \text{PV} \times 1.0147058\ldots^{10}$ ***(A1)***
OR
N= 10
I= 1.47058…
FV= 4000
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for all entries correct.

THEN
(PV =) \$3457 ***A1***

[3 marks]

(b) **METHOD 1** – *(Finding the future value of the investment using PV from part (a))*
N= 10
I=3.5
PV= 3446.66…(from Method 1) **OR** 3456.67…(from Methods 2, 3)
P/Y= 1
C/Y= 1 ***(M1)***

Note: Award ***M1*** for interest rate 3.5 **and** answer to part (a) as PV.

(FV=) \$4861.87 **OR** \$4875.97 ***(A1)***
so payment required (from TVM) will be \$294 **OR** \$295 ***A1***

Note: Award ***A0*** if a negative sign given, unless already penalized in part (a).

METHOD 2 – *(Using FV)*
N= 10
I=3.5
PV= -1000
FV= 4875.977…
P/Y= 1
C/Y= 1 ***(A1)(M1)***

Note: Award ***A1*** for I=3.5 **and** FV= $\pm 4875.977\ldots$, ***M1*** for all other entries correct **and** opposite PV and FV signs.

(PMT =) \$295 (295.393) ***A1***

Note: Correct 3sf answer is 295, however accept an answer of 296 given that the context supports rounding up. Award ***A0*** if a negative sign given, unless already penalized in part (a).

[3 marks]
Total [6 marks]

12. (a) $\mathrm{P}(\text{Type I error}) = \mathrm{P}(\text{stating female when male})$

$= \mathrm{P}(W_{Male} > 11.5)$ ***(M1)***

$= 0.00135 \quad (0.00134996\ldots)$ ***A1***

[2 marks]

(b) $\mathrm{P}(\text{Type II error}) = \mathrm{P}(\text{stating male when female})$

$= \mathrm{P}(W_{Female} < 11.5)$ ***(M1)***

$= 0.309 \quad (0.308537\ldots)$ ***A1***

[2 marks]

(c) attempt to use the total probability ***(M1)***

$\mathrm{P}(\text{error}) = 0.9 \times 0.00134996\ldots + 0.1 \times 0.308537\ldots$

$= 0.0321 \quad (0.0320687\ldots)$ ***A1***

[2 marks]

Total [6 marks]

13. (a) **METHOD 1**

recognizing that the real part is distributive ***(M1)***

$V_T = \mathrm{Re}\left(2\mathrm{e}^{3t\mathrm{i}} + 5\mathrm{e}^{3t\mathrm{i}+4\mathrm{i}}\right)$

$= \mathrm{Re}\left(\mathrm{e}^{3t\mathrm{i}}\left(2 + 5\mathrm{e}^{4\mathrm{i}}\right)\right)$ ***(A1)***

(from the GDC) $2 + 5\mathrm{e}^{4\mathrm{i}} = 3.99088\ldots\mathrm{e}^{-1.89418\ldots\mathrm{i}}$ ***(A1)***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$).

therefore $V_T = 3.99\cos(3t - 1.89) \quad \left(3.99088\ldots\cos(3t - 1.89418\ldots)\right)$ ***A1***

Note: Award the last ***A1*** for the correct values of A, B and C seen either in the required form or not. If method used is unclear and answer is partially incorrect, assume Method 2 and award appropriate marks eg. ***(M1)A1A0A0*** if only A value is correct.

continued…

Question 13 continued

METHOD 2

converting given expressions to cos form ***(M1)***

$V_T = 2\cos 3t + 5\cos(3t+4)$

(from graph) $A = 3.99$ $(3.99088\ldots)$ ***A1***

$V_T = 3.99\cos(Bt+C)$

either by considering transformations or inserting points

$B = 3$ ***A1***

$C = -1.89$ $(-1.89418\ldots)$ ***A1***

Note: Accept arguments differing by 2π e.g. $4.38900\ldots$.

(so, $V_T = 3.99\cos(3t-1.89)$ $\left(3.99088\ldots\cos(3t-1.89418\ldots)\right)$)

Note: It is possible to have $A = 3.99$, $B = -3$ with $C = 1.89$ **OR** $A = -3.99$, $B = 3$ with $C = 1.25$ **OR** $A = -3.99$, $B = -3$ with $C = -1.25$ due to properties of the cosine curve.

[4 marks]

(b) maximum voltage is 3.99 $(3.99088\ldots)$ (units) ***A1***

[1 mark]

Total [5 marks]

14. $V = \pi\int_0^{10} y^2\,\mathrm{d}x$ **OR** $\pi\int_0^{10} x^2\,\mathrm{d}y$ ***(M1)***

$h = 2$

$\approx \pi\times\frac{1}{2}\times 2\times\left(\left(4^2+5^2\right)+2\times\left(6^2+8^2+7^2+3^2\right)\right)$ ***M1A1***

$= 1120\text{ cm}^3$ $(1121.548\ldots)$ ***A1***

Note: Do not award the second ***M1*** If the terms are not squared.

Total [4 marks]

15. (a) (one vector to the line is $\begin{pmatrix}0\\c\end{pmatrix}$ therefore) $\boldsymbol{a}=\begin{pmatrix}0\\c\end{pmatrix}$ ***A1***

the line goes m up for every 1 across

(so the direction vector is) $\boldsymbol{b}=\begin{pmatrix}1\\m\end{pmatrix}$ ***A1***

Note: Although these are the most likely answers, many others are possible.

[2 marks]

(b) (from GDC **OR** $6\times 2-4\times 3$) $|M|=0$ ***A1***

[1 mark]

(c) **METHOD 1**

$\begin{pmatrix}X\\Y\end{pmatrix}=\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}x\\mx+c\end{pmatrix}=\begin{pmatrix}6x+3mx+3c\\4x+2mx+2c\end{pmatrix}$ ***M1A1***

$=\begin{pmatrix}3(2x+mx+c)\\2(2x+mx+c)\end{pmatrix}$ ***A1***

therefore the new line has equation $3Y=2X$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

METHOD 2

take two points on the line, e.g $(0,\ c)$ and $(1,\ m+c)$ ***M1***

these map to $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}0\\c\end{pmatrix}=\begin{pmatrix}3c\\2c\end{pmatrix}$

and $\begin{pmatrix}6&3\\4&2\end{pmatrix}\begin{pmatrix}1\\m+c\end{pmatrix}=\begin{pmatrix}6+3m+3c\\4+2m+2c\end{pmatrix}$ ***A1***

therefore a direction vector is $\begin{pmatrix}6+3m\\4+2m\end{pmatrix}=(2+m)\begin{pmatrix}3\\2\end{pmatrix}$

(since $m\neq -2$) a direction vector is $\begin{pmatrix}3\\2\end{pmatrix}$

the line passes through $\begin{pmatrix}3c\\2c\end{pmatrix}-c\begin{pmatrix}3\\2\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix}$ therefore it always has the origin as a jump-on vector ***A1***

the vector equation is therefore $\boldsymbol{r}=\mu\begin{pmatrix}3\\2\end{pmatrix}$ ***A1***

which is independent of m or c ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

continued…

Question 15 continued

METHOD 3

$$\boldsymbol{r} = \begin{pmatrix} 6 & 3 \\ 4 & 2 \end{pmatrix}\left(\begin{pmatrix} 0 \\ c \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ m \end{pmatrix}\right) = \begin{pmatrix} 3c \\ 2c \end{pmatrix} + \lambda \begin{pmatrix} 6+3m \\ 4+2m \end{pmatrix}$$ ***M1A1***

$$= c\begin{pmatrix} 3 \\ 2 \end{pmatrix} + (2+m)\lambda \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$ ***A1***

$$= \mu \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

where $\mu = c + (2+m)\lambda$ is an arbitrary parameter. ***A1***

which is independent of m or c (as μ can take any value) ***AG***

Note: The ***AG*** line (or equivalent) must be seen for the final ***A1*** line to be awarded.

[4 marks]

Total [7 marks]

16. (a) attempt at chain rule ***(M1)***

$$\left(\boldsymbol{v} = \frac{\mathrm{d}OP}{\mathrm{d}t} =\right) \begin{pmatrix} 2t\cos t^2 \\ -2t\sin t^2 \end{pmatrix}$$ ***A1***

[2 marks]

(b) attempt at product rule ***(M1)***

$$\boldsymbol{a} = \begin{pmatrix} 2\cos t^2 - 4t^2\sin t^2 \\ -2\sin t^2 - 4t^2\cos t^2 \end{pmatrix}$$ ***A1***

METHOD 1

let $S = \sin t^2$ and $C = \cos t^2$

finding $\cos\theta$ using

$$\boldsymbol{a} \bullet \overrightarrow{\mathrm{OP}} = 2SC - 4t^2S^2 - 2SC - 4t^2C^2 = -4t^2$$ ***M1***

$$|\overrightarrow{\mathrm{OP}}| = 1$$

$$|\boldsymbol{a}| = \sqrt{\left(2C - 4t^2S\right)^2 + \left(-2S - 4t^2C\right)^2}$$

$$= \sqrt{4 + 16t^4} > 4t^2$$

if θ is the angle between them, then

$$\cos\theta = -\frac{4t^2}{\sqrt{4+16t^4}}$$ ***A1***

so $-1 < \cos\theta < 0$ therefore the vectors are never parallel ***R1***

continued…

Question 16 continued

METHOD 2

solve

$$\begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \end{pmatrix} = k\begin{pmatrix} \sin t^2 \\ \cos t^2 \end{pmatrix}$$ ***M1***

then

$$(k =)\ \frac{2\cos t^2 - 4t^2 \sin t^2}{\sin t^2} = \frac{-2\sin t^2 - 4t^2 \cos t^2}{\cos t^2}$$

Note:	Condone candidates not excluding the division by zero case here. Some might go straight to the next line.

$$2\cos^2 t^2 - 4t^2 \cos t^2 \sin t^2 = -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2$$

$$2\cos^2 t^2 + 2\sin^2 t^2 = 0$$

$$2 = 0$$ ***A1***

this is never true so the two vectors are never parallel ***R1***

METHOD 3

embedding vectors in a 3d space and taking the cross product: ***M1***

$$\begin{pmatrix} \sin t^2 \\ \cos t^2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 2\cos t^2 - 4t^2 \sin t^2 \\ -2\sin t^2 - 4t^2 \cos t^2 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -2\sin^2 t^2 - 4t^2 \cos t^2 \sin t^2 - 2\cos^2 t^2 + 4t^2 \cos t^2 \sin t^2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$$ ***A1***

since the cross product is never zero, the two vectors are never parallel ***R1***

[5 marks]

Total [7 marks]

17. (a) use of chain rule ***(M1)***

$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$

attempt to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ at $x=1$ ***(M1)***

$0.2 = 0.04 \times \frac{\mathrm{d}x}{\mathrm{d}t}$

$\left(\frac{\mathrm{d}x}{\mathrm{d}t} =\right) 5 \text{ m h}^{-1}$ ***A1***

[3 marks]

(b) (i) if the position of the snail is (X, Y)

from part (a) $\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{1}{0.04X}\frac{\mathrm{d}Y}{\mathrm{d}t}$

since speed is 1:

finding modulus of velocity vector and equating to 1 ***(M1)***

$1 = \sqrt{\left(\frac{\dot{Y}}{0.04X}\right)^2 + \dot{Y}^2}$ **OR** $1 = \sqrt{\dot{X}^2 + 0.0016X^2\dot{X}^2}$

$1 = \dot{Y}^2\left(\frac{1}{0.0016X^2} + 1\right)$ **OR** $1 = \dot{X}^2(1 + 0.0016X^2)$

$\dot{Y} = \sqrt{\frac{1}{\frac{1}{0.08Y} + 1}}$ **OR** $\dot{X} = \sqrt{\frac{1}{1 + 0.0016X^2}}$ ***(A1)***

$\int_{0.02}^{2}\sqrt{\frac{1}{0.08Y} + 1}\ \mathrm{d}Y = \int_0^T \mathrm{d}t$ **OR** $\int_1^{10}\sqrt{1 + 0.0016X^2}\ \mathrm{d}X = \int_0^T \mathrm{d}t$ ***(M1)***

$T = 9.26$ hours ***A1***

(ii) **EITHER**

time for water to reach top is $\frac{2}{0.2} = 10$ hours (seen anywhere) ***A1***

OR

or at time $t = 9.26$, height of water is $0.2 \times 9.26 = 1.852$ ***A1***

THEN

so the water will not reach the snail ***AG***

[5 marks]

Total [8 marks]